SUMMARY
Pseudo-Smarandache Functions of First and Second kind*

A. S. Muktibodh †
Mohota Science College, Umred Rd.
Nagpur, India.

S.T. Rathod ‡
Mohota Science College, Umred Rd.
Nagpur,India.

In this paper we define two kinds of Pseudo-Smarandache functions. We have investigated more than fifty terms of each pseudo-Smarandache function. We have proved some interesting results and properties of these functions. The Pseudo-Smarandache function $Z(n)$ was introduced by Kashihara [4] as follows

1 Some definitions

Definition 1.1. *For any integer* $n \geq 1, Z(n)$ *is the smallest positive integer* m *such that* $1 + 2 + 3 + \ldots m$ *is divisible by* n.

The main results and properties of Pseudo-Smarandache functions are available in [3] [4],[5]. In the following we define Pseudo-Smarandache functions of first kind and second kind.

*This work is supported by UGC under the project No. 47-993/09 (WRO)
†E-mail: amukti2000@yahoo.com
‡Email: satish.rathod12@yahoo.com

Definition 1.2. *For any integer $n \geq 1$, the Pseudo-Smarandache function of first kind, $Z_1(n)$ is the smallest positive integer m such that $1^2+2^2+3^2 \ldots + m^2$ is divisible by n.*

Definition 1.3. *For any integer $n \geq 1$, the Pseudo-Smarandache function of second kind, $Z_2(n)$ is the smallest positive integer m such that $1^3 + 2^3 + 3^3 \ldots + m^3$ is divisible by n.*

2 Some Results for Pseudo-Smarandache functions of first kind.

Following results can be directly verified from the table given above.

1. $Z_1(n) = 1$ only if $n = 1$.

2. $Z_1(n) \geq 1$ for all $n \in N$.

3. $Z_1(p) \leq p$, where p is a prime.

4. If $Z_1(p) = n$, $p \neq 3$, then $p > n$.

Lemma 2.1. *If p is a prime then $Z_1(p) = p + 1$, for $p = 2$ or 3. Also, $Z_1(p) = \dfrac{p-1}{2}$ for $p \geq 5$.*

Lemma 2.2. *For $p = 2$, $Z_1(p^k) = p^{k+1} - 1$.*

Lemma 2.3. $Z_1(n) \geq max\{Z_1(N) : N \mid n\}$.

Lemma 2.4. *Let $n = \dfrac{k(k+1)(2k+1)}{6}$ for some $k \in N$, then $Z_1(n) = k$.*

Lemma 2.5. *It is not possible that $Z_1(m) = m$ for any $m \in N$.*

Lemma 2.6. *$S(m) = k$ then $S(m) = Z_1(2k+1)$.*

3 Some Results on Pseudo-Smarandache function of second kind

Following properties are result of direct verification.

1. $Z_2(n) = n$ only for $n = 1$.
2. $Z_2(p) = p - 1$, $p \neq 2$. $Z_2(p) = p + 1$ for $p = 2$.
3. $Z_2(n) \geq max\{Z_2(N) : N \mid n\}$.

Following are some of the important results.

Lemma 3.1. *If $S(n) = k$ then $Z_2(k) = n$.*

Open Problem: What is the relation between $Z_1(n)$ and $Z_2(n)$?

References

[1] Aschbacher Charles, On Numbers that are Pseudo Smarandache and Smarandache Perfect, Smarandache Notions Journal , 14(2004), p.p. 40-41.

[2] Gioia, Anthony A. The Theory of Numbers- An Introduction, NY, U.S.A. Dover Publications Inc., 2001.

[3] Ibstedt, Henry, Surfing on the Ocean of Numbers- A Few Smarandache Notions and Similar Topics, U.S.A. Erhus University Press, 1997.

[4] Kashihara, Kenichiro, Comments and topics on Smarandache Notions and problems, U.S.A. Erhus University Press, 1996.

[5] A.A.K.Majumdar, A note on Pseudo Smarandache Function , Scientia Magna, Vol. 2, (2006), No. 3, 1-25.